7.5 Roots and Zeros

Fundamental Theorem of Algebra

- all polynomial equations with a degree greater than zero have at least one root in the set of complex numbers
- a polynomial equation of the form P(x) = 0 of degree n with complex coefficients, has exactly n roots

How many roots?

ex:

a - 10 = 0 1

$x^2 + 2x - 48 = 0$ 2

$3a^3 + 18a = 0$ 3

$y^5 - y = 0$ 5

Solve # 3.

$3a(a^2 + 6) = 0$

$3a = 0$ $\quad a^2 + 6 = 0$

$a = 0$ $\quad a^2 = -6$

$a = \pm i\sqrt{6}$

1 ℝ
2 imag

Solve # 4.

$y^5 - y = 0$

$y(y^4 - 1) = 0$

$y(y^2 + 1)(y^2 - 1) = 0$

$y = 0$ $\quad y^2 = -1$ $\quad y^2 = 1$

3 ℝ
2 imag $\quad y = \pm i$ $\quad y = \pm 1$

Complex Conjugates

a, b $\in$ R $\quad b \neq 0$

If a + bi is a zero, then a - bi is a zero

1. If 3 + 2i is a zero, then __3 – 2i___ is also a zero.

2. If 5 - 3i is a zero, then __5+3i___ is also a zero.

3. If $1 - \sqrt{5}$ is a zero, then __$1+\sqrt{5}$___ is also a zero. (It also works for radicals)

4. If 6i is a zero, then ___-6i___ is also a zero.

Write a function with the following zeros.

3, 2 + i , 2 − i

sum = 4

prod = 4 − i² = 5

(x − 3)(x² − 4x + 5) = 0

x³ − 7x² + 17x − 15 = 0

Write a function with the following zeros.

-2, $2-\sqrt{3}$, $2+\sqrt{3}$

sum = 4

prod = 1 4 − 3

(x + 2)(x² − 4x + 1) = 0

x³ − 4x² + x

+2x² − 8x + 2

x³ − 2x² − 7x + 2 = 0

Write a function with the following zeros.

-8, 5i −5i

5 = 0

P = −25i² = 25

(x + 8)(x² + 25) = 0

HW
p375-376
13, 15, 17,
35, 38, 39, 41